Lecture Notes
in Computational Science
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45

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Dimension Reduction of Large-Scale Systems

Proceedings of a Workshop held in Oberwolfach, Germany, October 19–25, 2003

With 95 Figures and 29 Tables

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Preface

This volume is a result of the mini workshop *Dimension Reduction of Large-Scale Systems* which took place at the Mathematisches Forschungsinstitut Oberwolfach, Germany, October 19–25, 2003. The purpose was to bring together experts from different communities and application areas in an attempt to synthesize major ideas in dimension reduction that have evolved simultaneously but separately in several areas involving simulation and control of complex physical processes. The systems that inevitably arise in such simulations are often too complex to meet the expediency requirements of interactive design, optimization, or real time control. Model order reduction has been devised as a means to reduce the dimensionality of these complex systems to a level that is amenable to such requirements.

Model order reduction seeks to replace a large-scale system of differential or difference equations by a system of substantially lower dimension that has nearly the same response characteristics. Dimension reduction is a common theme within the simulation and control of complex physical processes. Generally, large systems arise due to accuracy requirements on the spatial discretization of control problems for fluids or structures, in the context of lumped-circuit approximations of distributed circuit elements, such as the interconnect or package of VLSI chips. Dimension reduction is generally required for purposes of expediency and/or storage reduction. Applications can be found in

- Simulation of conservative systems, e.g., in Molecular Dynamics,
- Control and regulation of fluid flow (CFD),
- Simulation and stabilization of large structures,
- Control design for (land, air, sea) vehicles,
- VLSI chip design,
- Simulation of micro-electro-mechanical systems (MEMS),
- Semiconductor simulations,
- Image processing,

and many other areas.

Various reduction techniques have been devised, but many of these are described in terms that are discipline-oriented or even application-specific even though they share many common features and origins. This workshop was aimed at bringing together specialists from several fields and application areas in order to expose the similarities of these approaches, to identify common features, to address application-specific challenges, and to investigate how successful reduction methods for linear systems might be applied to nonlinear dynamic systems and very large scale problems with state-space dimensions of order in the millions.

The problems in dimension reduction are challenging from the mathematical and algorithmic points of view. For example, the selection of appropriate basis functions in reduced-order basis approaches like proper orthogonal decomposition (POD) is highly problem-specific and requires a deeper mathematical understanding. On the algorithmic side there is a clear need for additional work in the area of large scale numerical linear algebra. Moreover, it is of considerable interest to introduce some non-traditional techniques such as wavelet bases.

Methods with global computable error bounds are missing in almost all application areas except for medium-size control problems. Here, Gramian-based methods (e.g., balanced truncation) have been successfully applied to approximating the input-output behavior of linear systems and *a posteriori* error bounds can be easily computed. For very large-scale problems or systems based on differential-algebraic equations (DAEs), it is not yet clear how to apply these techniques. For very large scale problems, advanced numerical linear algebra techniques are needed to address the huge matrix dimensions and difficulties resulting, e.g., from irregular sparsity patterns as in circuit simulation. For the special DAE systems arising, e.g., in circuit simulation, methods based on partial realization (moment matching or Padé approximation) have been developed. Though they are successful in some areas, they still lack global error bounds and have difficulties when special system properties such as stability or passivity are to be preserved by the reduced-order model.

During the workshop there were presentations on a variety of theories and methods associated with the above mentioned applications. With this book, we wish to give an overview of the range of topics and to generate interest in

- analyzing the available methods and mathematical theory,
- extracting the best features from different methods,
- developing a deeper mathematical understanding of the methods and application-specific challenges,
- combining good features and new mathematical ideas with the goal of designing superior methods.

A goal of the workshop and this book is to describe some of the most prominent approaches, to discuss common features and point out issues in need of further investigation. We hope to stimulate a broader effort in the area of order reduction for large-scale systems that will lead to new mathematical and algorithmic tools with the ability to tackle challenging problems in scientific computing ranging from control of nonlinear PDEs to the DC analysis of future generation VLSI chips.

An equally important aspect to this workshop is the collection and distribution of an extensive set of test problems and application specific benchmarks. This should make it much easier to develop relevant methods and to systematically test them.

The participants (in alphabetical order) were Athanasios C. Antoulas (Rice University, Houston, USA), Zhaojun Bai (University of California at Davis, USA), Peter Benner (TU Chemnitz, Germany), Roland W. Freund (Bell Laboratories, Murray Hill, USA), Serkan Gugercin (Virginia Tech, Blacksburg, USA), Michael Hinze (TU Dresden, Germany), Jing-Rebecca Li (INRIA, Rocquencourt, France), Karl Meerbergen (FFT, Leuven, Belgium), Volker Mehrmann (TU Berlin, Germany), Danny C. Sorensen (Rice University, Houston, USA), Tatjana Stykel (TU Berlin, Germany), Paul Van Dooren (Université Catholique de Louvain, Belgium), Andras Varga (DLR Oberpfaffenhofen, Germany), Stefan Volkwein (Universität Graz, Austria), and as a visitor for one day, Jan Korvink (IMETK, University of Freiburg, Germany).

The lively discussions inside this group really inspired this effort to write a collection of articles serving as tutorials to a general audience in the same spirit of the talks as they were presented during the workshop. The decision to provide a set of benchmark examples that should serve as test cases in the development and evaluation of new algorithms for model and dimension reduction was also a product of these discussions. We, the organizers, wish to thank the participants and we hope that the wider research community will find this effort useful.

We would like to thank the Mathematisches Forschungsinstitut Oberwolfach for providing the possibility to organize this Mini-workshop on Dimension Reduction. This opportunity and the fantastic research environment has made this initiative possible.

Chemnitz, Berlin, Houston
February 2005

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Contents

Part I Papers

Part I

Papers

The first and main part of this book contains ten papers that are written by the participants of the Oberwolfach mini-workshop *Dimension Reduction of Large-Scale Systems*. In most parts, they are kept in a tutorial style in order to allow non-experts to get an overview over some major ideas in current dimension reduction methods. The first 4 papers (Chapters 1–4) discuss various aspects of balancing-related techniques for large-scale systems, structured systems, and descriptor systems. Model reduction techniques for time-varying systems are presented in Chapter 5. The next three papers (Chapters 6–8) treat model reduction for second- and higher-order systems, which can be considered as one of the major research directions in dimension reduction for linear systems. Chapter 9 discusses controller reduction techniques—here, large-scale has a somewhat different meaning than in classical model reduction as controllers are considered as "large" already when the number of states describing the controller's dynamics exceeds 10. The last paper in this part (Chapter 10) concentrates on proper orthogonal decomposition—currently probably the mostly used and most successful model reduction technique for nonlinear systems.

We hope that the surveys on current trends presented here can be used as a starting point for research in dimension reduction methods and stimulates discussions on improving and extending the currently available approaches.