A New Test for Passivity of Descriptor Systems

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Outline

- Descriptor systems
- Passive systems
- The positive real lemma
- Testing positive realness
- Conclusions and outlook

Linear Descriptor Systems

Linear time-invariant systems in generalized state-space form:

$$
\begin{aligned}
E\dot{x}(t) &= Ax(t) + Bu(t), \\
y(t) &= Cx(t) + Du(t),
\end{aligned}
$$

arise, e.g., in

- control and simulation of coupled systems,
- control of multibody (mechanical) systems,
- manipulation of fluid flow (e.g., semi-discretized Navier-Stokes equations),
- circuit simulation, VLSI chip design, in particular modeling of interconnet via RLC networks,
- simulation of MEMS and NEMS (micro-/nano-electro-mechanical systems).

Assumptions

- n generalized states / descriptor variables, i.e., $x(t) \in \mathbb{R}^n$;
- m "inputs", i.e., $u(t) \in \mathbb{R}^m$;
- m "outputs", i.e., $y(t) \in \mathbb{R}^m$;
- $A - \lambda E$ regular, i.e., $\exists \lambda \in \mathbb{C} : \det(A - \lambda E) \neq 0$;
- $A - \lambda E$ stable, i.e., $\lambda(A, E) \subset \mathbb{C}^- \cup \{\infty\}$ $\Rightarrow$ system is stable.

Corresponding transfer function:

$$G(s) = C(sE - A)^{-1}B + D$$

Weierstraß Canonical Form and Index

Regular matrix pencils are equivalent ($(A, E) \mapsto (PAQ, PEQ)$) to their

Weierstraß canonical form (WCF) $\left[\begin{array}{cc} J & 0 \\ 0 & I_{n_\infty} \end{array}\right] - \lambda \left[\begin{array}{cc} I_{n_f} & 0 \\ 0 & N \end{array}\right].$

Here:

- J contains Jordan blocks corresponding to the $n_f = n - n_\infty$ finite eigenvalues,
- N is nilpotent and contains Jordan blocks to the n_∞ infinite eigenvalues,
- $\nu :=$ size of largest Jordan block in N is called the (algebraic) index of $A - \lambda E$.

Fast and Slow Subsystems

Matrix pencil equivalence implies restricted system equivalence (RSE):

$$(E, A, B, C, D) \sim (PEQ, PAQ, PB, CQ, D) \text{ for any nonsingular } P, Q \in \mathbb{R}^{n \times n}.$$

With the coordinate transformation $x \mapsto Px =: \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and a corresponding partitioning

$$PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad CQ = \begin{bmatrix} C_1 & C_2 \end{bmatrix},$$

RSE and the WCF imply the decoupling

$$\left.\begin{array}{rcl} \dot{x}_1 & = & Jx_1 + B_1 u, \\ y_1 & = & C_1 x_1 + Du \end{array}\right\} \text{ slow subsystem}$$

$$\left.\begin{array}{rcl} N\dot{x}_2 & = & x_2 + B_2 u, \\ y_2 & = & C_2 x_2 \end{array}\right\} \text{ fast subsystem}$$

Additive Decomposition of Transfer Function

Partial information about a descriptor system is obtained from its block-diagonal form

$$S(A-\lambda E)T = \begin{bmatrix} A_f & \\ & A_\infty \end{bmatrix} - \lambda \begin{bmatrix} E_f & \\ & E_\infty \end{bmatrix}, \quad SB = \begin{bmatrix} B_f \\ B_\infty \end{bmatrix}, \; CT = \begin{bmatrix} C_f & C_\infty \end{bmatrix},$$

where $S, T \in \mathbb{R}^{n\times n}$ are nonsingular. This yields

slow-fast decoupling:

$$\left.\begin{array}{rcl} E_f \dot{x}_f &=& A_f x_f + B_f u, \\ y_f &=& C_f x_f + Du \end{array}\right\} \quad \text{slow subsystem}$$

$$\left.\begin{array}{rcl} E_\infty \dot{x}_\infty &=& A_\infty x_\infty + B_\infty u, \\ y_\infty &=& C_\infty x_\infty \end{array}\right\} \quad \text{fast subsystem}$$

additive decomposition of $G(s)$:

$$G(s) = G_f(s) + G_\infty(s) = C_f(sE_f - A_f)^{-1}B_f + D + C_\infty(sE_\infty - A_\infty)^{-1}B_\infty.$$

Markov Parameters

A rational transfer function $G(s) = C(sE - A)^{-1}B + D$ has a power series expansion (Laurent series) at $s_0 = \infty$ of the form

$$G(s) = \sum_{k=-\infty}^{q} s^k M_k,$$

where $M_k \in \mathbb{R}^{m \times m}$ are the Markov parameters of G and $q \leq \nu$.

This implies

$$G(s) = G_{sp}(s) + M_0 + sM_1 + \sum_{k=2}^{q} s^k M_k = G_p(s) + sM_1 + \sum_{k=2}^{q} s^k M_k,$$

where

- $G_{sp}(s)$ is strictly proper, i.e., $\lim_{s\to\infty} G_{sp}(s) = 0$;
- $G_p(s) := G_{sp} + M_0$ is proper, i.e., $\lim_{s\to\infty} G_p(s)$ is finite.

Passive Systems

Definition:
A system is passive if $\int_{-\infty}^{t} u(\tau)^T y(\tau)\, d\tau \geq 0 \;\forall t \in \mathbb{R},\ \forall u \in L_2(\mathbb{R}, \mathbb{R}^m)$.

"The system cannot generate energy."

Anderson/Vongpanitlerd 1973:

Theorem: system is passive $\Longleftrightarrow$ its transfer function is positive real

Definition:
A *real*, rational matrix-valued function $G : \mathbb{C} \to \overline{\mathbb{C}}^{m \times m}$ is (strictly) positive real if

1. G is analytic in $\mathbb{C}^+ := \{s \in \mathbb{C} \mid \mathrm{Re}(s) > 0\}$,
2. $G(s) + G^T(\bar{s}) \geq 0$ for all $s \in \mathbb{C}^+$ ($G(s) + G^T(\bar{s}) > 0$ for all $s \in \overline{\mathbb{C}^+}$.)

Motivation

Guaranteeing passivity of descriptor systems is necessary in

Model reduction for passive systems:

- Task often encountered in circuit simulation, VLSI chip design.
- Padé-type methods in general do not preserve passivity, post-processing necessary [Bai/(Feldmann)/Freund '98,'01].
  - PRIMA [Odabasioglu et al.'96,'97] preserves passivity for interconnect models, basically Arnoldi process.
  - SyPVL preserves passivity for RLC circuits [Feldmann/Freund '96,'97].
  - LR-ADI/dominant subspace approximation can preserve passivity [Li/White '01].
- But in general: passivity not guaranteed for reduced-order models computed by Padé-type methods, needs to be checked!

Model Reference Adaptive Control:

passivity of "input-to-tracking error" transfer function guarantees tracking property [Landau 1979, Dai 1989, ...].

Hybrid Methods for Model Reduction

Recent "trend" in model reduction of LARGE-scale systems (MEMS, CFD):
"LARGE" $= \; n > 100,000.$

Apply method for large sparse eigenproblems like

- Jacobi-Davidson,
- Krylov-subspace method (Lanczos, Arnoldi),

to compute projector onto low-dimensional ($n \approx 1000$) subspace using

- modal truncation,
- Padé(-type) approximation,

ideas, then reduce the intermediate model further by balancing-related techniques.

For passive systems, need to check that intermediate system is still passive.

The Positive Real Lemma

KALMAN/YAKUBOVICH/POPOV/ANDERSON 1962–67

Theorem:
Let (A, B, C, D) be a minimal realization of a linear time-invariant system with transfer function $G(s)$.
a) $G(s)$ is positive real $\Longleftrightarrow$ $\exists$ a solution $X \geq 0$ to the LMI

$$\begin{bmatrix} A^T X + XA & XB - C^T \\ B^T X - C & -(D + D^T) \end{bmatrix} \leq 0.$$

b) If $D + D^T > 0$, then $G(s)$ is strictly positive real $\Longleftrightarrow$
the algebraic Riccati equation (ARE)

$$A^T X + XA + (XB - C^T)(D + D^T)^{-1}(B^T X - C) = 0,$$

has a stabilizing solution X.

A Positive Real Lemma for Descriptor Systems

Freund/Jarre 2000/2004

Theorem:

a) (Sufficiency) Let (E, A, B, C, D) be a realization of a linear descriptor system with transfer function $G(s)$. If the LMIs

$$\begin{bmatrix} A^T X + X^T A & X^T B - C^T \\ B^T X - C & -(D + D^T) \end{bmatrix} \leq 0, \quad E^T X = X^T E \geq 0$$

have a solution X, then $G(s)$ is positive real.

b) (Necesssity) Let (E, A, B, C, D) be a minimal realization of a linear descriptor system with transfer function $G(s)$ satisfying

$$D + D^T \geq M_0 + M_0^T, \quad \text{where } M_0 \text{ is the 0th Markov parameter of } G.$$

Then, if $G(s)$ is positive real, there exists a solution of the LMIs given above.

Testing Positive Realness of Descriptor Systems

- Testing positive realness via LMIs often not feasible due to computational complexity ($\mathcal{O}(n^6)$, employing structure $\mathcal{O}(n^5)$).

- Even in case $D + D^T > 0$, the Riccati equation/Hamiltonian eigenproblem test is not applicable if E is singular.

- Eigenvalue-based test for scalar transfer functions of standard and descriptor systems exists, but no generalization to MIMO is known. [Bai/Freund 2000]

- For standard systems, recursive reduction procedure can be applied: $(A, B, C, D) \rightarrow (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ with $\tilde{D} + \tilde{D}^T$ nonsingular; $G(s)$ is then positive real if $\tilde{G}(s)$ is strictly positive real. [Weiss/Wang/Speyer 1994]

Goal: algebraic test for positive realness that only requires orthogonal coordinate transformations and has complexity $\mathcal{O}(n^3)$.

Markov Parameters and Positive Realness

ANDERSON/VONGPANITLERD 1973, FREUND/JARRE 2000:

Theorem: Given a rational matrix-valued function

$$G(s) = \underbrace{G_p(s)}_{\text{proper}} + sM_1 + \sum_{k=2}^{q} s^k M_k,$$

then $G(s)$ is positive real $\iff \begin{cases} 1. & G_p(s) \text{ is positive real,} \\ 2. & M_1 \geq 0, \\ 3. & M_k = 0, \; k = 2, 3, \ldots, q. \end{cases}$

Remarks:

Condition 3. is trivially satisfied if the index of $A - \lambda E$ satisfies $\nu \leq 2$.

Conditions 2.+3. are trivially satisfied if $\nu \leq 1$.

Method I

Algorithm:

1. Compute an additive decomposition of $G(s)$,

$$\begin{aligned} G(s) &= G_f(s) + G_\infty(s) \\ &= \left[C_f(sE_f - A_f)^{-1}B_f + D\right] + \left[C_\infty(sE_\infty - A_\infty)^{-1}B_\infty\right]. \end{aligned}$$

2. Test if $G_\infty(s) = -C_\infty A_\infty^{-1} B_\infty - sC_\infty A_\infty^{-1} E_\infty A_\infty^{-1} B_\infty$.
3. Test if $-C_\infty A_\infty^{-1} E_\infty A_\infty^{-1} B_\infty$ is positive semidefinite.
4. Test if $G_f(s) - C_\infty A_\infty^{-1} B_\infty$ is positive real.

Remarks:

- Step 1. may be ill-conditioned as non-orthogonal transformations are rquired.
- Step 2. could be tested using sufficiently many interpolation points s_k so that $s_k E - A$ is well-conditioned.
- Method I is feasible for index-1 descriptor systems as Steps 2.+3. are redundant and Step 1. is “easier”.

Method II

Main ideas:

- assume minimality (otherwise, compute minimal realization first, using e.g., MATLAB Descriptor Systems Toolbox [VARGA 2000–2005]).
- avoid additive decomposition, instead compute new reduced form of $A - \lambda E$ using only orthogonal equivalence transformations;
- from reduced form obtain explicit expressions for $G_p(s)$, M_1, and M_k for $k = 2, \ldots, q$;
- the conditions $M_k = 0$ for $k \geq 2$ can be checked via the rank indices of the new reduced form;
- testing $M_1 \geq 0$ by adopted algorithm employing orthogonal decompositions only (complexity $\mathcal{O}(n^3)$);
- testing $G_p(s)$ positive real using reduction to "strictly positive real" check.

Method II: Orthogonal Reducing Equivalence Transformation

Lemma:

For any regular pencil $A - \lambda E$ there exist orthogonal matrices $U, V \in \mathbb{R}^{n\times n}$ such that

$$U(A-\lambda E)V = \begin{array}{c} \\ \end{array}\begin{array}{cccc} n_1 & n_2 & n_3 & n_4 \end{array}$$

$$U(A-\lambda E)V = \begin{bmatrix} A_{11}-\lambda E_{11} & A_{12}-\lambda E_{12} & A_{13}-\lambda E_{13} & A_{14}-\lambda E_{14} \\ 0 & A_{22} & A_{23}-\lambda E_{23} & A_{24}-\lambda E_{24} \\ 0 & 0 & A_{33} & A_{34} \\ 0 & 0 & 0 & A_{44} \end{bmatrix} \begin{array}{l} \}n_1 \\ \}n_3 \\ \}n_2 \\ \}n_4 \end{array},$$

where $\operatorname{rank}(E_{11}) = n_1, \quad \operatorname{rank}(E_{23}) = n_3, \quad \operatorname{rank}(A_{44}) = n_4$, and

$$\operatorname{rank}\left(\begin{bmatrix} A_{22} & A_{23}-\lambda E_{23} \\ 0 & A_{33} \end{bmatrix}\right) = n_2 + n_3 \ \forall \lambda \in \mathbb{C}.$$

Proof: constructive algorithm using 3 URVs (SVDs), 1 RQ factorization, 1 generalized Schur decomposition $\rightsquigarrow$ complexity $\mathcal{O}(n^3)$.

Method II: Reducing Equivalence Transformation

Note: this step is only needed for theoretical purposes, no computations erquired!

Due to partitioning of transformed matrix pencil, as an intermediate step to WCF we can find nonsingular matrices

$$X = \begin{bmatrix} I_{n_1} & X_2 & X_3 & X_4 \\ 0 & I_{n_3} & 0 & 0 \\ 0 & 0 & I_{n_2} & 0 \\ 0 & 0 & 0 & I_{n_4} \end{bmatrix}, \quad Y = \begin{bmatrix} I_{n_1} & Y_2 & Y_3 & Y_4 \\ 0 & I_{n_2} & 0 & 0 \\ 0 & 0 & I_{n_3} & 0 \\ 0 & 0 & 0 & I_{n_4} \end{bmatrix},$$

such that

$$XU(A - \lambda E)VY = \begin{bmatrix} A_{11} - \lambda E_{11} & 0 & 0 & 0 \\ & A_{22} & A_{23} - \lambda E_{23} & A_{24} - \lambda E_{24} \\ & & A_{33} & A_{34} \\ & & & A_{44} \end{bmatrix}.$$

Method II: Representation of G_p and M_k

After the RSE transformation implied by XU, VY we get

$$G(s) = C_1(sE_{11} - A_{11})^{-1}(B_1 + X_2B_2 + X_3B_3 + X_4B_4) + D$$
$$+ \begin{bmatrix} C_2 + C_1Y_2 & C_3 + C_1Y_3 & C_4 + C_1Y_4 \end{bmatrix} \begin{bmatrix} A_{22} & -sE_{23} + A_{23} & -sE_{24} + A_{24} \\ 0 & A_{33} & A_{34} \\ 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} B_2 \\ B_3 \\ B_4 \end{bmatrix}.$$

$\Longrightarrow$ formulae for G_p, M_k:

$$G_p(s) = C_1(sE_{11} - A_{11})^{-1}(B_1 + X_2B_2 + X_3B_3 + X_4B_4) + D$$
$$- \begin{bmatrix} C_2 + C_1Y_2 & C_3 + C_1Y_3 & C_4 + C_1Y_4 \end{bmatrix} \begin{bmatrix} A_{22} & A_{23} & A_{24} \\ 0 & A_{33} & A_{34} \\ 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} B_2 \\ B_3 \\ B_4 \end{bmatrix},$$

$$M_k = - \begin{bmatrix} C_2 + C_1Y_2 & C_3 + C_1Y_3 & C_4 + C_1Y_4 \end{bmatrix} \left(\begin{bmatrix} A_{22} & A_{23} & A_{24} \\ 0 & A_{33} & A_{34} \\ 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} 0 & E_{23} & E_{24} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)^k \times$$
$$\times \begin{bmatrix} A_{22} & A_{23} & A_{24} \\ 0 & A_{33} & A_{34} \\ 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} B_2 \\ B_3 \\ B_4 \end{bmatrix}, \qquad k = 1, \ldots, n_2 + n_3.$$

Method II: Simplification of G_p, M_k

E_{23} nonsingular $\Longrightarrow$ for $k = 1, \cdots, n_2 + n_3$

$$M_k = -\left[\begin{array}{cc} C_2 + C_1 Y_2 & C_3 + C_1 Y_3 \end{array}\right] \left(\left[\begin{array}{cc} A_{22} & A_{23} \\ 0 & A_{33} \end{array}\right]^{-1} \left[\begin{array}{cc} 0 & E_{23} \\ 0 & 0 \end{array}\right] \right)^k \left[\begin{array}{cc} A_{22} & A_{23} \\ 0 & A_{33} \end{array}\right]^{-1} \times$$

$$\times \left(\left[\begin{array}{c} B_2 \\ B_3 \end{array}\right] - \left[\begin{array}{c} A_{24} - A_{23} E_{23}^{-1} E_{24} \\ A_{34} - A_{33} E_{23}^{-1} E_{24} \end{array}\right] A_{44}^{-1} B_4 \right).$$

With this, the transfer function becomes

$$G(s) = \hat{C} \left(s \left[\begin{array}{ccc} E_{11} & & \\ & 0 & E_{23} \\ & 0 & 0 \end{array}\right] - \left[\begin{array}{ccc} A_{11} & & \\ & A_{22} & A_{23} \\ & 0 & A_{33} \end{array}\right] \right)^{-1} \hat{B} + \hat{D},$$

where with notation from WCF

$$E_{11} \sim I_{n_f}, \quad A_{11} \sim J, \quad \left[\begin{array}{cc} 0 & E_{23} \\ 0 & 0 \end{array}\right] \sim N, \quad \left[\begin{array}{cc} A_{22} & A_{23} \\ 0 & A_{33} \end{array}\right] \sim I_{n_\infty}.$$

Method II: Testing $M_k = 0$ for $k = 2, \ldots, q$

Lemma:

a) $n_2 = n_3 \Longrightarrow M_k = 0$ for $k = 2, \cdots, n_2 + n_3$.

b) $G(s)$ positive real $\Longrightarrow n_2 = n_3$.

Proof:

a) $n_2 = n_3 \;\Rightarrow\; \begin{bmatrix} A_{22} & A_{23} \\ 0 & A_{33} \end{bmatrix}^{-1} = \begin{bmatrix} A_{22}^{-1} & * \\ 0 & A_{33}^{-1} \end{bmatrix} \;\Rightarrow\; \left(\begin{bmatrix} A_{22} & A_{23} \\ 0 & A_{33} \end{bmatrix}^{-1} \begin{bmatrix} 0 & E_{23} \\ 0 & 0 \end{bmatrix} \right)^k = 0 \;\forall\, k \geq 2.$

b) [Freund/Jarre]: G minimal, positive real $\Rightarrow$ nilpotent matrix N from WCF

- is either empty $\Rightarrow\; n_2 = n_3 = 0$;
- or has only 2×2 Jordan blocks $\Longrightarrow n_2 + n_3$ even.

  Now, $N \sim \begin{bmatrix} 0 & E_{23} \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & I_{n_3} \\ 0 & 0 \end{bmatrix}$, with one "1" from each Jordan block $\Rightarrow\; n_2 = n_3$.

Method II: Case $n_3 = 0$

$A - \lambda E$ regular $\Longrightarrow$ $n_2 = n_3 = 0$ $\Longrightarrow$ $M_1 = 0$ and

$$\begin{aligned} G_p(s) &= G(s) = \begin{bmatrix} C_1 & C_4 \end{bmatrix} \left(s \begin{bmatrix} E_{11} & E_{14} \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A_{11} & A_{14} \\ 0 & A_{44} \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1 \\ B_4 \end{bmatrix} + \mathcal{D}. \\ &= \begin{bmatrix} \mathcal{C}_1 & \mathcal{C}_2 \end{bmatrix} \left(s \begin{bmatrix} \mathcal{E}_{11} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathcal{B}_1 \\ \mathcal{B}_2 \end{bmatrix} + \mathcal{D}, \end{aligned}$$

with $\mathcal{E}_{11}, \mathcal{A}_{22}$ nonsingular.

Thus, from transformed form of $A - \lambda E$ positive realness test for $G(s)$ is reduced to testing positive realness of proper transfer function.

Method II: Case $n_2 = n_3 \neq 0$

$$\operatorname{rank}\left(\begin{bmatrix} A_{22} & A_{23} - \lambda E_{23} \\ 0 & A_{33} \end{bmatrix}\right) = n_2 + n_3 \ \forall \lambda \in \mathbb{C} \implies A_{22} \text{ and } A_{33} \text{ nonsingular.}$$

$\implies$ New representation of G_p and M_1:

$$G_p(s) = \begin{bmatrix} 0 & 0 & C_1 & C_2 & C_3 & C_4 \end{bmatrix} \left(s \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{23} & E_{24} \\ 0 & 0 & E_{11} & E_{12} & E_{13} & E_{14} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right.$$

$$\left. - \begin{bmatrix} -E_{11} & -E_{12} & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 & 0 & 0 \\ A_{11} & A_{12} & A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & 0 & 0 & A_{22} & A_{23} & A_{24} \\ 0 & 0 & 0 & 0 & A_{33} & A_{34} \\ 0 & 0 & 0 & 0 & 0 & A_{44} \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} C_1 & C_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & 0 & 0 \\ 0 & -A_{22} & E_{23} & E_{24} \\ 0 & 0 & -A_{33} & -A_{34} \\ 0 & 0 & 0 & -A_{44} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ B_3 \\ B_4 \end{bmatrix}.$$

Method II: Case $n_2 = n_3 \neq 0$

$M_1 \geq 0$**:** using a sequence of RQ/QR/QR factorizations, we get

$$M_1 = \mathcal{N}^{-1}\mathcal{M}, \quad \mathcal{N}, \mathcal{M} \in \mathbb{R}^{m \times m}.$$

$\Longrightarrow$ $M_1 \geq 0 \quad \Longleftrightarrow \quad \mathcal{M}\mathcal{N}^T \geq 0$

$G_p(s)$ **positive real:** using a sequence of QR/RQ factorizations, we get

$$G_p(s) = \left[\begin{array}{cc} \mathcal{C}_1 & \mathcal{C}_2 \end{array}\right] \left(s \left[\begin{array}{cc} \mathcal{E}_{11} & 0 \\ 0 & 0 \end{array}\right] - \left[\begin{array}{cc} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{array}\right] \right)^{-1} \left[\begin{array}{c} \mathcal{B}_1 \\ \mathcal{B}_2 \end{array}\right] + \mathcal{D},$$

with $\mathcal{E}_{11}, \mathcal{A}_{22}$ nonsingular.

$\Longrightarrow$ Positive realness of $G_p(s)$ reduced to positive realness test for proper transfer function in standard form.

Conclusions and Outlook

- $\mathcal{O}(n^3)$ numerical algorithm for reducing the passivity test for descriptor systems to passivity test for minimal, proper transfer functions, using orthogonal RSE only.

- Based on the special form of proper systems to be tested for positive realness, we also have derived a special recursive orthogonal reduction procedure to ARE-based "strict positive realness" test.

- Implementation and thorough numerical testing necessary.

- Usage of a priori knowledge about, e.g., index or nullspace of E, can improve the performance of the algorithms.

- Method I suitable if index-1 or properness can be assumed.

- None of the approaches applicable to large, sparse descriptor systems, but sufficient for testing positive realness in model reduction methods based on modal truncation and Padé-type approximation.

- Sometimes, Writing down an LMI is not the end in computational control.