Parametric Model Reduction

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Background

Model order reduction (MOR) is a common theme within the simulation, control and optimization of complex physical processes.
However, significant modifications to the underlying physical model, such as

- geometric variations,
- changes in material properties,
- alterations in boundary conditions

should be preserved in the reduced-order system. Thus, new methods for parametric model order reduction (PMOR) are required.
Consider a linear parametric system

$$E\dot{x}(t) = \sum_{i=1}^{d} p_i A_i x(t) + Bu(t), \quad y(t) = Cx(t),$$

where $E, A_i \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$.

The goal of PMOR is to obtain an accurate, reduced, parametric model

$$\hat{E}\dot{\hat{x}}(t) = \sum_{i=1}^{d} p_i \hat{A}_i \hat{x}(t) + \hat{B}u(t), \quad \hat{y}(t) = \hat{C}\hat{x}(t),$$

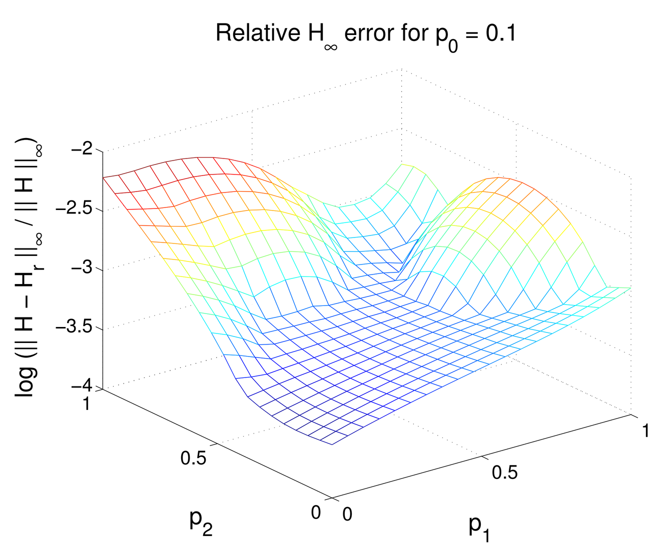
with $\hat{E}, \hat{A}_i \in \mathbb{R}^{q\times q}$, $\hat{B} \in \mathbb{R}^{q\times m}$, and $\hat{C} \in \mathbb{R}^{p\times q}$, which not only has much less degrees of freedom $q \ll n$, but also preserves all parameters.

Interpolatory Methods for PMOR

Derivation of a unifying projection-based framework for interpolatory PMOR with

- preservation of structural parameter dependence (linear or nonlinear),
- matching gradient and Hessian of the system response at interpolation points,
- optimal choice of interpolation data for special SISO parameterizations.

Results



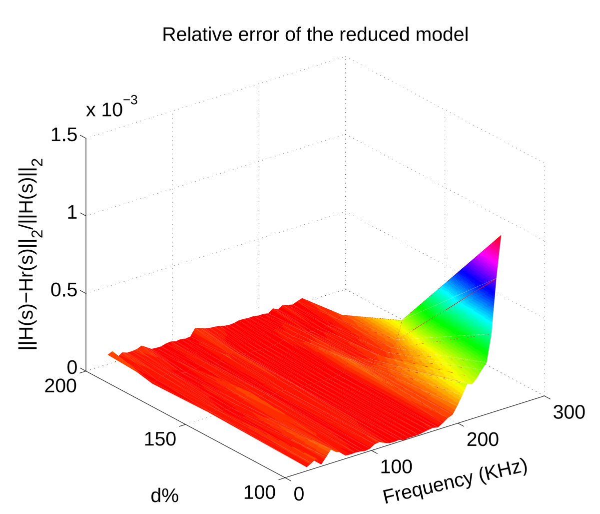
- $\mathcal{H}_2$-optimal rational interpolation.
- Thermal compact model with 3 parameters.
- Dimension reduced from 4257 to 14.

Automatic PMOR for Microsystem Simulation

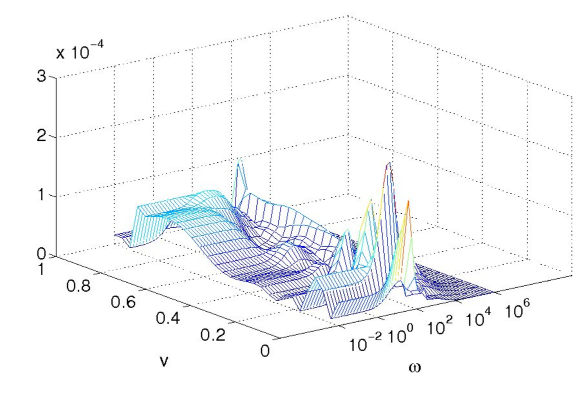
In current design processes for complex microsystems, numerical simulation plays an important role. We developed two different approaches: the first one is an implicit multimoment-matching method, the second depends on balanced truncation. Both approaches are coupled with rational interpolation and have the following properties:

- preservation of parameter dependence (linear, nonlinear, time varying),
- efficient computation of the reduced model,
- integration into design process for microsystems.

Results



- Implicit multi-moment matching PMOR.
- Multi-moments are matched.
- Reduced model of Gyroscope with 2 geometrical parameters.
- Dimension reduced from 17913 to 235.



- Balanced truncation/rational interpolation.
- Anemometer example, 1 parameter.
- Absolute $\mathcal{H}_\infty$ error.
- Use of sparse grids for higher dimensional parameter spaces.
- Dimension reduced from 29008 to 75.

PMOR via Bilinear MOR

In this project, we consider linear parameter-varying (LPV) systems of the form

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{d} p_i(t) A_i x(t) + B_0 u_0(t), \quad y(t) = Cx(t),$$

where $A, A_i \in \mathbb{R}^{n\times n}$, $B_0 \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$.

The advantage is that there exists a close connection to bilinear control systems

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{m} N_i x(t) u_i(t) + Bu(t), \quad y(t) = Cx(t),$$

where $A, N_i \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$.

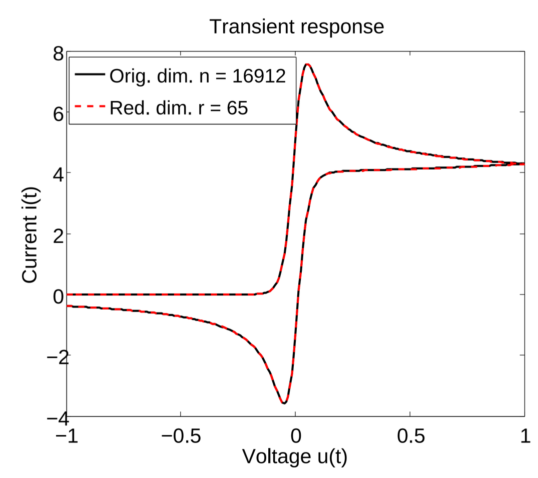
Further information

- Parameters are incorporated in a nonlinear system and are automatically preserved.
- Balanced truncation relies on generalized Lyapunov equations.
- Quality of a reduced model $\hat{\Sigma}$ can be measured by bilinear $\mathcal{H}_2$-norm:

$$||\hat{\Sigma}||^2_{\mathcal{H}_2} := \sum_{k=1}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \frac{1}{(2\pi)^k} \overline{H_k(i\omega_1,\dots,i\omega_k)} H_k^T(i\omega_1,\dots,i\omega_k),$$

where H_k denote generalized transfer functions of the system.

Results



- Simulation of a cyclic voltammogram.
- Time-varying system with non-zero initial condition.
- Dimension reduced from 16912 to 65.

Cooperation

Prof. Beattie, Prof. Gugercin, Virginia Tech., Blacksburg, USA

Prof. Korvink, University of Freiburg, Freiburg, Germany

Prof. Willcox, Massachusetts Institute of Technology, USA

Future Directions

- PMOR for nonlinear parametric systems from chemical, biological engineering.
- Optimal choice of interpolation points for more general parametric systems.
- PMOR for coupled systems.

References

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