DFG-SCHWERPUNKTPROGRAMM 1253
OPTIMIZATION WITH PARTIAL DIFFERENTIAL EQUATIONS

BAD HONNEF 06. AND 07. 02. 2006

Optimal control-based feedback stabilization of multi-field flow problems

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Objectives

- Derive and investigate numerical algorithms for optimal control-based boundary feedback stabilization of multi-field flow problems.
- Explore the potentials and limitations of feedback-based (Riccati) stabilization techniques.
- Employ recent advances in reducing complexity of Riccati solvers to achieve stabilization cost proportional to the simulation of the forward problem.
- Extend current methods for flow described by Navier-Stokes equations to flow problems coupled with other field equations of increasing complexity.

Context

- Method-oriented, numerical analysis and implementation of structure-exploiting algorithms;
- Riccati-based stabilization so far mostly used for smaller scales — here, new algorithms will allow application to discretized problems of complexity 10^6–10^{10};
- Optimal control for coupled PDEs (relevant model problems of exemplary character for several engineering applications).

Previous work by others

- Stabilization of flows (with velocity field v and pressure p), described by Navier-Stokes equations (NSE)

$$\partial_t v + v\cdot\nabla v - \frac{1}{Re}\Delta v + \nabla p = f \text{ in } \Omega\times(0,\infty)$$
$$\operatorname{div} v = 0,$$

  $\Omega \subseteq \mathbb{R}^d$, $d = 2, 3$, to steady-state solutions.
- Existence of stabilizing feedback control proved in 2D [Fernández-Cara et al 2004] and 3D [Fursikov 2004].
- Construction based on associated linear-quadratic optimal control problem:
  - for distributed control, see [Barbu 2003, Barbu/Sritharan 1998, Barbu/Triggiani 2004];
  - for boundary control, see [Barbu/Lasiecka/Triggiani 2005, Raymond 2005].

References

[Fernández-Cara et al 2004] Y. Fernández-Cara, S. Guerrero, O.Yu. Imanuvilov, and J.-P. Puel. Local exact controllability of the Navier-Stokes system. J. Math. Pures Appl., IX. Sér., 83(12):1501–1542, 2004.

[Fursikov 2004] A.V. Fursikov. Stabilization for the 3D Navier-Stokes system by feedback boundary control. Discrete Contin. Dyn. Syst., 10(1–2):289–314, 2004.

[Barbu 2003] V. Barbu. Feedback stabilization of the Navier-Stokes equations. ESAIM: Control Optim. Calc. Var., 9:197–206, 2003.

[Barbu/Lasiecka/Triggiani 2005] V. Barbu, I. Lasiecka, and R. Triggiani. Boundary stabilization of Navier-Stokes equations. Memoirs of the AMS, to appear.

[Barbu/Sritharan 1998] V. Barbu and S. S. Sritharan. H^∞-control theory of fluid dynamics. Proc. R. Soc. Lond. A, 454:3009–3033, 1998.

[Barbu/Triggiani 2004] V. Barbu and R. Triggiani. Internal stabilization of Navier-Stokes equations with finite dimensional controllers. Indiana Univ. Math. J., 53(5):1443–1494, 2004.

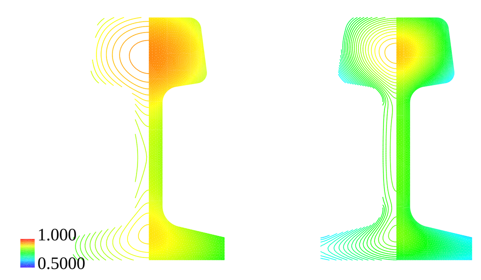
[Raymond 2005] J.-P. Raymond. Local boundary feedback stabilization of the Navier-Stokes equations. In Control Systems: Theory, Numerics and Applications, Rome, 30 March – 1 April 2005, Proceedings of Science. SISSA, http://pos.sissa.it, 2005.

Previous work of applicants

Optimal control-based stabilization for nonlinear PDEs:

- Optimal control and (Riccati-based) stabilization for linear and nonlinear (convection-)diffusion equations, e.g., nonlinear heat equation for optimal cooling of steel profiles:

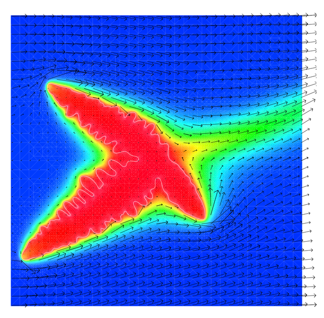
$$c(\Theta)\rho(\Theta)\Theta_t = \nabla.(\lambda(\Theta)\nabla\Theta) \quad \text{in} \quad \Omega\times[0,t_f],$$
$$\lambda(\Theta)\partial_\nu\Theta = \alpha(\Theta-\Theta_{ext}) + \beta(\Theta^4-\Theta_{ext}^4)$$



- Large sparse (algebraic and differential) Riccati solver (in progress): modular concept allows application to a large class of PDEs (through interfaces to PDE simulation software).

Computational Fluid Dynamics:

- Simulation of coupled flow problems and free surface flows;
- Applications to materials science, flows under micro-gravity conditions, crystal growth, non-Newtonian fluids etc.;
- Code NAVIER:
  - FEM code for the simulation of laminar, transient, incompressible flows, e.g. transient melt-flow during crystal growth:



  - Functionality: coupling with advection equations, Stefan problems, free surface flows, free capillary surfaces and other "non-standard" boundary conditions;
  - 2d, 2.5d, 3d versions;
  - Operator splitting based on fractional step θ-scheme.

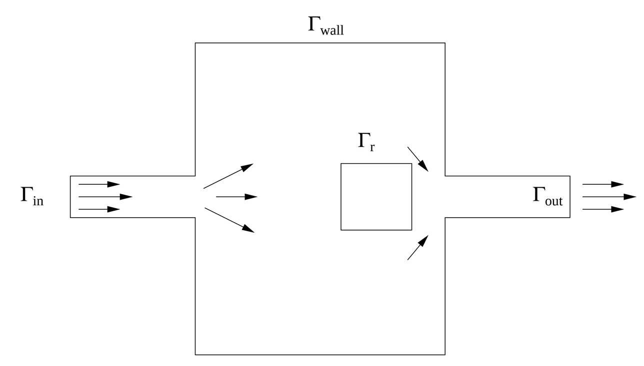
Methodological Tasks

1. Derive a linearized equation for the controlled system.
2. Set up an associated linear-quadratic optimal control problem and the associated operator Riccati equation.
3. Find a suitable spatial Galerkin FEM discretization of the linearized control system and the associated algebraic Riccati equations (ARE).
4. Derive a problem-dependent (structure exploiting) algorithm for solving the resulting ARE. Requires combination of Newton-ADI with efficient solvers for the associated stationary PDE. (Emphasis on efficient preconditioners.)
5. Apply the resulting finite-dimensional feedback control to the original nonlinear PDE problem and verify its stabilization properties.

Extensive computational studies for each of the scenarios will be performed with the goal to obtain numerical evidence of the stabilization properties of the feedback law as well as of the convergence properties to the infinite-dimensional control.

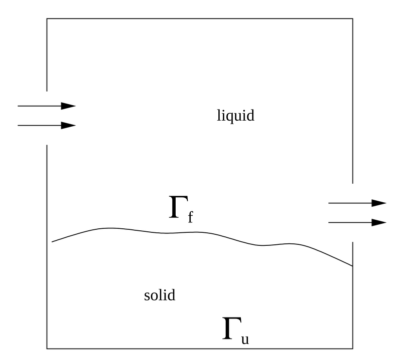
Application Tasks

1. NSE with passive transport of a reactive species.



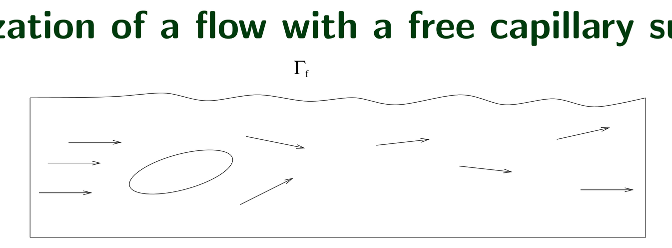
Setup: Reactive species is transported from $\Gamma_{\mathbf{in}}$ to reacting surface Γ_r.
Aim: Stabilize and control the reaction.
Control: Inflow velocity distribution v at $\Gamma_{\mathbf{in}}$.
⇝ NSE coupled with linear convection-diffusion equation.

2. Phase transition liquid/solid with convection.



Setup: Hot melt that solidifies while flowing through a mould.
Aim: Control the phase boundary Γ_f between the liquid and solid parts.
Control: Temperature distribution on parts of boundary.
⇝ NSE coupled with Stefan problem for free boundary.

3. Stabilization of a flow with a free capillary surface.



Setup: The flow past the obstacle is oscillatory due to Karman vortex shedding.
Aim: Stabilize the capillary free surfaces Γ_f.
Control: Inflow velocity distribution.
⇝ NSE coupled with highly nonlinear (boundary) curvature operator.

Envisioned cooperation

Within SPP1253:

- with Michael Hinze (TU Dresden) on control of flow problems, in particular alternative stabilization schemes for NSE;
- with Fredi Tröltzsch (TU Berlin) on analytical aspects of existence and regularity of solutions;
- with Lars Grüne (Universität Bayreuth) on the influence of discrete implementation of feedback control for PDEs.

Internationally:

- with Jean-Pierre Raymond (Université Paul Sabatier, Toulouse) on the construction of linear-quadratic optimal control problems associated to flow problems;
- Andy Wathen (University of Oxford) on the construction of efficient preconditioners for solving linearized problems.