The Hamiltonian Extended Krylov Subspace Method

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An algorithm for constructing a J-orthogonal basis of the extended Krylov subspace $\mathcal{K}_{r,s} = \text{range}\{u, Hu, H^2u, \ldots, H^{2r-1}u, H^{-1}u, H^{-2}u, \ldots\ldots, H^{-2s}u\}$, where $H \in \mathbb{R}^{2n\times 2n}$ is a large (and sparse) Hamiltonian matrix is derived (for $r = s+1$ or $r = s$). Surprisingly, this allows for short recurrences involving at most five previously generated basis vectors. Projecting H onto the subspace $\mathcal{K}_{r,s}$ yields a small Hamiltonian matrix. The resulting HEKS algorithm may be used in order to approximate $f(H)u$, where f is a function which maps the Hamiltonian matrix H to, e.g., a (skew-)Hamiltonian or symplectic matrix. Numerical experiments illustrate that approximating $f(H)u$ with the HEKS algorithm is competitive for some functions compared to the use of other (structure-preserving) Krylov subspace methods.

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