

Abstract ID : 17

Recent Advances in the Solution of the Unilateral Quadratic Matrix Equation

Content

The Unilateral Quadratic Matrix Equation (UQME)

$$\begin{equation} AX^2 + BX + C = 0 \end{equation}$$

$$\end{equation}$$

is a rarely mentioned equation in numerical linear algebra. Only a few publications and algorithms cover the solution of this equation. In this talk, we will see some existing solvers, namely the Schur-Method, the Bernoulli-Iteration, the Newton-Method, and the Cyclic Reduction. All these solvers have issues that avoid a general usage. Especially bad or missing convergences is crucial at this point. One approach to get a general purpose solver is to use a spectral divide and conquer scheme to solve the underlying eigenvalue problem

$$\begin{equation} Fy = \begin{bmatrix} 0 & I \\ -C & -B \end{bmatrix} y = \lambda Gy, \quad y = \begin{bmatrix} v \\ \lambda v \end{bmatrix}. \end{equation}$$

$$Fy = \begin{bmatrix} 0 & I \\ -C & -B \end{bmatrix} y = \lambda Gy, \quad y = \begin{bmatrix} v \\ \lambda v \end{bmatrix}.$$

$$\end{bmatrix} y = \lambda Gy, \quad y = \begin{bmatrix} v \\ \lambda v \end{bmatrix}.$$

$$\end{bmatrix} y = \lambda Gy, \quad y = \begin{bmatrix} v \\ \lambda v \end{bmatrix}.$$

$$y = \lambda \begin{bmatrix} v \\ \lambda v \end{bmatrix}$$

$$I \otimes 0 \oplus A \oplus \begin{bmatrix} y \\ \lambda y \end{bmatrix} = \lambda Gy, \quad y = \begin{bmatrix} v \\ \lambda v \end{bmatrix}$$

$$v \oplus \lambda v$$

$$\end{bmatrix}.$$

$$\end{equation}$$

By computing the generalized Schur decomposition of $(F, G) = Q(T, S)Z^H$ we obtain

$$\begin{equation} Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}, \end{equation}$$

$$\end{equation}$$

$$\end{equation}$$

from which the solution X of the UQME can be constructed via

$$\begin{equation} X = Z_{21} Z_{11}^{-1}. \end{equation}$$

$$\end{equation}$$

$$\end{equation}$$

The spectral divide and conquer algorithm successively constructs the matrix Z until Z_{11} and Z_{12} can be extracted. The presentation will describe the new spectral division approach and compares it to the existing ones.

Primary author: KÖHLER, Martin (Max Planck Institute for Dynamics of Complex Technical Systems)

Presenter: KÖHLER, Martin (Max Planck Institute for Dynamics of Complex Technical Systems)

Status: SUBMITTED