A survey on computational methods for Lyapunov-plus-Positive equations

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We consider linear matrix equations of the form

$$AXE^T + EXA^T + \sum_{i=1}^{m} N_i X N_i^T = -Y \tag{1}$$

which are called generalized Lyapunov-plus-Positive equations. These equations come up in model order reduction of bilinear dynamical systems of the form

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + \sum_{i=1}^{m} N_i x(t) u(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

where $A, E \in \mathbb{R}^{n \times n}$, $N_i \in \mathbb{R}^{n \times n}$ $\forall i \in \{1, ..., m\}$, $B \in \mathbb{R}^{n \times r}$, $C \in \mathbb{R}^{p \times n}$ and $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^r$ is the system input and $y(t) \in \mathbb{R}^p$ is the system output. The controllability and observability Gramians P and Q of the system are used in model order reduction methods like balanced truncation to obtain a reduced system. The Gramians can be computed through the following equations of type (1):

$$\begin{aligned} APE^T + EPA^T + \sum_{i=1}^{m} N_i P N_i^T &= -BB^T, \\ A^T QE + E^T QA + \sum_{i=1}^{m} N_i^T Q N_i &= -C^T C. \end{aligned}$$

In this setting, the coefficient matrices are often large and sparse. Solutions to (1) are generally dense and too large to fit into memory. Thus there is a need for solvers that compute low-rank approximations of the solution.

Multiple approaches to computing solutions of (1) have recently emerged. In this talk, we review the most promising approaches and discuss their upsides and limitations. Finally, we compare the approaches in practice through implementations using the in-house software library C-MESS. Most of the presented methods can also be applied to more general multiterm linear matrix equations.

This presentation is an intermediate report of my master thesis project.