Max-Planck-Institut für
Dynamik komplexer technischer Systeme
Max Planck Institute for Dynamics of Complex Technical Systems

Thesis Topic

Research Group
"Computational Methods in Systems and Control Theory"

Title

"Kalman-Yakubovič-Popov Lemma for Algebraic Difference Equations"

Job Description

We consider the algebraic difference equation

$$Ex_{k+1} = Ax_k + Bu_k, \quad x_0 = x^0,$$

with E, $A \in \mathbb{K}^{n\times n}$, $B \in \mathbb{K}^{n\times m}$, generalized state vector $x_k \in \mathbb{K}^n$ and control input vector $u_k \in \mathbb{K}^m$. The matrix E might be singular and therefore the dynamics of x_k underlies additional hidden algebraic constraints.

In various applications, one often considers rational matrix functions (so called Popov functions or spectral density functions) of the form

$$\Phi(z) = \begin{bmatrix} (\overline{z}E - A)^{-1}B \\ I_m \end{bmatrix}^* \begin{bmatrix} Q & S \\ S^* & R \end{bmatrix} \begin{bmatrix} (z^{-1}E - A)^{-1}B \\ I_m \end{bmatrix}, \tag{1}$$

with $Q = Q^*$ and $R = R^*$. Properties of such functions are often related to the solvability of certain linear matrix inequalities (LMIs).

The first task consists of analyzing a conditions for the equivalence between

$$\Phi(e^{\mathrm{i}\omega}) \geq 0 \quad \text{for all} \quad e^{\mathrm{i}\omega} \notin \Lambda(E, A) \tag{2}$$

and the solvability of the LMI

$$\begin{bmatrix} A^*XA - E^*XE + Q & A^*XB + S \\ B^*XA + S^* & B^*XB + R \end{bmatrix} \geq 0, \quad X = X^*.$$

Similar results obtained for the differential algebraic equation case should be adapted to this problem.

The second part of the thesis is devoted to the characterization of (2) via the eigenstructure of the palindromic matrix pencil

$$\lambda \begin{bmatrix} 0 & E & 0 \\ A^* & Q & S \\ B^* & S^* & R \end{bmatrix} - \begin{bmatrix} 0 & A & B \\ E^* & Q & S \\ 0 & S^* & R \end{bmatrix}. \tag{3}$$

Dipl.-Math. Matthias Voigt

Computational Methods in Systems and Control Theory

Phone: +49 391 6110 464
Fax: +49 391 6110 500

Email:
voigtm@mpi-magdeburg.mpg.de

www:
http://www.mpi-magdeburg.mpg.de/mpcsc/voigtm/

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For this purpose, properties of the palindromic Kronecker canonical form of the pencil should be analyzed. This step also requires to deal with certain transformations of (1) and (3).

Optionally, different applications of this theory can be taken into account. These include the characterization of certain structural properties of dynamical systems via solvability of LMIs and the eigenstructure of associated palindromic pencils. Furthermore, one might consider the relation of these concepts to the optimal control problem

$$\min \sum_{k=0}^{\infty} \begin{bmatrix} x_k \\ u_k \end{bmatrix}^* \begin{bmatrix} Q & S \\ S^* & R \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} \quad \text{subject to} \quad Ex_{k+1} = Ax_k + Bu_k, \quad x_0 = x^0.$$

References

[1] Hassibi, B., Sayed, A.H., Kailath, T.: *Indefinite-Quadratic Estimation and Control: A Unified Approach to H^2 and H^∞ Theories*, SIAM Studies in Applied and Numerical Mathematics, Philadelphia, PA, 1999.

[2] Schröder, C.: *Palindromic and Even Eigenvalue Problems – Analysis and Numerical Methods*, PhD Thesis, TU Berlin, Apr. 2008.

[3] Reis, T.: *Lur'e equations and even matrix pencils*, Lin. Alg. Appl., 434(1), pp. 152–173, 2011.

[4] Reis, T., Voigt, M.: *Spectral Factorization for Differential-Algebraic Systems*, in preparation.

Job Requirements

Recommended: Systems and Control Theory,
Desirable: Complex Analysis, Numerical Linear Algebra, Matrix Equations.

Degree

Diplom or Master