Max-Planck-Institut für
Dynamik komplexer technischer Systeme
Max Planck Institute for Dynamics of Complex Technical Systems

MAX−PLANCK−INSTITUT FÜR DYNAMIK KOMPLEXER TECHNISCHER SYSTEME MAGDEBURG

Thesis Topic

Research Group
"Computational Methods in Systems and Control Theory"

Dipl.-Math. Matthias Voigt

Computational Methods in Systems and Control Theory

Phone: +49 391 6110 464
Fax: +49 391 6110 500

Email:
voigtm@mpi-magdeburg.mpg.de

www:
http://www.mpi-magdeburg.mpg.de/mpcsc/voigtm/

October 21, 2013

Title

"Fast Numerical Computation of Structured Real Stability Radii for Large-Scale Matrices and Pencils"

Job Description

Consider the continuous-time linear time-invariant descriptor system

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t). \end{aligned}$$

where E, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $x(t) \in \mathbb{R}^n$ is the descriptor vector, $u(t) \in \mathbb{R}^m$ is the input vector, and $y(t) \in \mathbb{R}^p$ is the output vector. Here, E is usually a singular matrix, so we obtain a differential-algebraic system. Systems of this kind are the natural formulation of many dynamical models arising, e.g., in the simulation, control and optimization of electrical circuits, constrained multi-body systems, or certain semidiscretized PDEs.

Of particular interest are questions concerning stability and robustness of such a system. Assume that the system is asymptotically stable, i.e., the finite eigenvalues of $\lambda E - A$, denoted by $\Lambda_{\mathrm{f}}(E, A)$, are in the open left halt-plane and all solutions of

$$E\dot{x}(t) = Ax(t)$$

converge to zero for $t \to \infty$ and all consistent initial conditions. This thesis project deals with the question: What is the smallest *real* perturbation matrix Δ such that the system

$$E\dot{x}(t) = (A + B\Delta C)x(t)$$

will be destabilized (i.e., one of the eigenvalues will move to the right half-plane)? The norm of the matrix Δ is then called structured real stability radius and denoted by $r_{\mathbb{R}}(E, A, B, C)$.

For systems with a small state-space dimension n, there already exist formulas and algorithms relying on dense matrix algebra to compute $r_{\mathbb{R}}(E, A, B, C)$. However, this is not the case if the system consists of large and sparse matrices E, A, B, and C.

To solve this problem one considers real structured pseudospectra

$$\Lambda_\varepsilon(E,A,B,C) := \big\{ s \in \mathbb{C} : s \in \Lambda_{\mathrm{f}}(E, A + B\Delta C) \text{ for some } \Delta \in \mathbb{R}^{m\times p} \\ \text{with } \|\Delta\| < \varepsilon \big\}.$$

To obtain $r_{\mathbb{R}}(E,A,B,C)$, one has to find the value of ε such that $\Lambda_\varepsilon(E,A,B,C)$ touches the imaginary axis. This can be done by a nested iteration. The inner iteration computes the rightmost pseudoeigenvalue for *a fixed value of* ε. To do this, some recently developed algorithms should be adapted. These use the fact, that an optimal perturbation of low rank that gives the rightmost pseudoeigenvalue, can be efficiently constructed. In the outer iteration, the value of ε is varied by employing Newton's algorithm.

Summarizing, the task of this project is to understand the real structured perturbation theory of the matrix pencils $\lambda E - A$ and to adapt the algorithms from the literature to compute $r_{\mathbb{R}}(E,A,B,C)$. Finally, the method should be implemented in MATLAB and tested using some benchmark examples.

References

[1] P. Benner and M. Voigt. *A structured pseudospectral method for $\mathcal{H}_\infty$-norm computation of large-scale descriptor systems*, Preprint MPIMD/12-10, Max Planck Institute Magdeburg, 2012.

[2] N. H. Du, V. H. Linh, and V. Mehrmann. *Robust Stability of Differential-Algebraic Equations*, in A. Ilchmann, T. Reis (eds.), *Surveys in Differential-Algebraic Equations I*, Differential-Algebraic Equations Forum, Springer-Verlag Berlin, Heidelberg, 2013.

[3] N. Guglielmi and C. Lubich. *Low-rank dynamics for computing extremal points of real pseudospectra*, SIAM J. Matrix Anal. Appl., 34(1):40–66, 2013.

[4] N. Guglielmi, M. Gürbüzbalaban, and M. L. Overton. *Fast approximation of the H_∞ norm via optimization over spectral value sets*, SIAM J. Matrix Anal. Appl., 34(2):709–737, 2013.

Job Requirements

Recommended: Differential-Algebraic Equations, Numerical Analysis, Systems and Control Theory.
Desirable: Complex Analysis, Differential Geometry, Numerical Linear Algebra (Eigenvalue Problems).

Degree

Diplom or Master